II. THE MASS GAP AND SOLUTION OF THE QUARK CONFINEMENT PROBLEM IN QCD

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We have investigated a closed system of equations for the quark propagator, obtained earlier within our general approach to QCD at low energies. It implies quark confinement (the quark propagator has no pole, indeed), as well as the dynamical breakdown of chiral symmetry (a chiral symmetry preserving solution is forbidden). This system can be solved exactly in the chiral limit. We have established the space of the smooth test functions (consisting of the Green's functions for the quark propagator and the corresponding quark-gluon vertex) in which our generalized function (the confining gluon propagator) becomes a continuous linear functional. It is a linear topological space $K(c)$ of the infinitely differentiable functions (with respect to the dimensionless momentum variable x), having compact support in the region $x \leq c$. We develop an analytical formalism, the so-called chiral perturbation theory at the fundamental quark level, which allows one to find explicit solution for the quark propagator in powers of the light quark masses. We also develop an analytical formalism, which allows one to find the solution for the quark propagator in the inverse powers of the heavy quark masses. It justifies the use for the heavy quark propagator its free counterpart up to terms of the order $1/m_Q^3$, where m_Q is the heavy quark mass. So this solution automatically possesses the heavy quark spin-flavor symmetry.

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I. INTRODUCTION

In our previous works [1, 2] we have formulated the intrinsically nonperturbative (INP) QCD as the true theory of QCD at low energies. In general, it is defined as QCD from which all types and at all level the perturbative (PT) contributions (”PT contaminations”) are to be subtracted. This theory makes it possible to calculate the physical observables/processes in low-energy QCD from first principles in a self-consistent way. One of the main roles in the realization of this program belongs to the solution for the gluon Green's function which describes their propagation in the QCD vacuum. In the presence of a mass gap responsible for the true NP QCD dynamics it has been exactly established [1]. In the subsequent paper [2] we apply this solution to the quark, ghost and ghost-quark sectors in order to derive the system of equations for the quark propagator, namely

$$\begin{aligned} S^{-1}(p) &= S_0^{-1}(p) + \Lambda_{NP}^2 \Gamma_\mu(p,0) S(p) \gamma_\mu, \\ \Gamma_\mu(p,0) &= i d_\mu S^{-1}(p) - S(p) \Gamma_\mu(p,0) S^{-1}(p). \end{aligned} \tag{1.1}$$

The obtained system of equations (1.1) is exact, i.e., no approximations/truncations have been made so far. Formally it is valid in the whole energy/momentum range, but depends only on the mass gap Λ_{NP}^2 responsible for the true NP QCD dynamics. It is free of all types of the PT contributions (”PT contaminations”) at the fundamental quark-gluon-ghost level. As mentioned in Ref. [2], the free PT quark propagator is not to be subtracted in order to maintain the chiral limit physics in QCD. This limit is important to correctly understand the structure of QCD at low energies. Also, it is manifestly gauge-invariant, i.e., does not depend explicitly on the gauge-fixing parameter.

The main purpose of this work is to solve this system explicitly and show that it describes the confining quark propagator, indeed. We will show that it possesses the property of the dynamical breakdown of chiral symmetry as well. Concluding, let us only note that a very preliminary derivation of the system of equations (1.1) has been made in our earlier papers [3, 4].

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II. SOLUTION

The simplest way to solve the ground system of equations (1.1) is to represent the corresponding proper quark-gluon vertex at zero momentum transfer as its decomposition into the four independent matrix structures, i.e.,

$$\Gamma_\mu(p,0) = \gamma_\mu F_1(p^2) + p_\mu F_2(p^2) + p_\mu \hat{p} F_3(p^2) + \hat{p}\gamma_\mu F_4(p^2). \tag{2.1}$$

The Euclidean version of the chosen parametrization for the full quark propagators is as follows:

$$iS(p) = \hat{p} A(p^2) - B(p^2), \tag{2.2}$$

so its inverse is

$$iS^{-1}(p) = \hat{p}\overline{A}(p^2) + \overline{B}(p^2), \tag{2.3}$$

where

$$\begin{aligned} \overline{A}(p^2) &= A(p^2)E^{-1}(p^2), \\ \overline{B}(p^2) &= B(p^2)E^{-1}(p^2), \\ E(p^2) &= p^2 A^2(p^2) + B^2(p^2). \end{aligned} \tag{2.4}$$

Substituting the representation (2.1) into the second equation of the system (1.1), on account of the previous relations, and doing some tedious algebra of the γ-matrices in the 4D Euclidean space we can express the scalar functions $F_i(p^2)$ $(i = 1, 2, 3, 4)$ in terms of the quark propagator form factors $A(p^2)$ and $B(p^2)$ as follows:

$$\begin{aligned} F_1(p^2) &= -\frac{1}{2}\overline{A}(p^2), \\ F_2(p^2) &= -\overline{B}'(p^2) + F_4(p^2), \\ F_3(p^2) &= -\overline{A}'(p^2), \\ F_4(p^2) &= -\frac{1}{2}\overline{A}^2(p^2)\overline{B}^{-1}(p^2). \end{aligned} \tag{2.5}$$

Here the prime denotes the derivative with respect to the Euclidean momentum variable p^2.

Substituting the solution for the ST identity (2.5) back into the quark SD equation, which is the first of the equations in the ground system (1.1), and again doing the above-mentioned rather tedious but well-known algebra and introducing further the dimensionless variables and functions

$$A(p^2) = \Lambda_{NP}^{-2} A(x), \qquad B(p^2) = \Lambda_{NP}^{-1} B(x), \qquad x = p^2/\Lambda_{NP}^2, \tag{2.6}$$

one finally obtains that the ground system of equations (1.1) is reduced to

$$\begin{aligned} xA' &= -(2+x)A - 1 - \bar{m}_0 B, \\ 2BB' &= -3A^2 - 2(B - \bar{m}_0 A)B. \end{aligned} \tag{2.7}$$

Here and everywhere below $A \equiv A(x)$, $B \equiv B(x)$, and the prime denotes the derivative with respect to the Euclidean dimensionless momentum variable x. We also introduce the notation for the dimensionless current quark mass as follows: $\bar{m}_0 = m_0/\Lambda_{NP}$, and omitting flavor index, for simplicity. As mentioned above, this system and consequently the initial system (1.1) for the first time has been obtained in Refs. [3, 4].

The formal exact solution of the ground system (2.7) for the dynamically generated quark mass function $B(x)$ is

$$B^2(c, \bar{m}_0; x) = \exp(-2x) \int_x^c \exp(2x')\tilde{\nu}(x')\, dx', \tag{2.8}$$

where c is the constant of integration. Not losing generality, it can be fixed as $c = p_c^2/\Lambda_{NP}^2$, where p_c^2 is some constant momentum squared. Also

$$\tilde{\nu}(x) = A(x)[3A(x) + 2\nu(x)] \tag{2.9}$$

with

$$\nu(x) = xA'(x) + (2+x)A(x) + 1 = -\bar{m}_0 B(x). \tag{2.10}$$

Then the equation for determining the $A(x)$ function becomes

$$\frac{d\nu^2(x)}{dx} + 2\nu^2(x) = -\tilde{\nu}(x)\bar{m}_0^2. \tag{2.11}$$

It is interesting to note that the last equation can be exactly solved up to the terms of the order $\bar{m}_0$, and may be even up to the terms of the order $\bar{m}_0^2$, since the function $\nu(x)$ is of the order $\bar{m}_0$ itself (see Eq. (2.10)).

A. Chiral limit

In the chiral limit ($\bar{m}_0 = 0$) the ground system (2.7) is reduced to

$$\begin{aligned} xA_0' &= -(2+x)A_0 - 1, \\ 2B_0B_0' &= -3A_0^2 - 2B_0^2, \end{aligned} \tag{2.12}$$

which can be solved exactly. The exact solution for the $A_0(x)$ function is

$$A_0(x) = \frac{1}{x^2}\left\{1 - x - \exp(-x)\right\}, \tag{2.13}$$

while for the dynamically generated quark mass function $B_0(x)$ the exact solution is

$$B_0^2(c_0; x) = 3\exp(-2x)\int_x^{c_0} \exp(2x')A_0^2(x')\,dx', \tag{2.14}$$

where $c_0 = p_0^2/\Lambda_{NP}^2$ is the constant of integration, and p_0^2 is some constant momentum squared for the chiral limit case. The solution (2.13) has the correct asymptotic properties. It is regular at small x and asymptotically approaches the free propagator at infinity. This behavior can formally be achieved in two ways: $p^2 \to \infty$ at fixed Λ_{NP}^2 and/or by $\Lambda_{NP}^2 \to 0$ as well. Let us remind that the last limit is known as the PT one. It is interesting to note that the chiral limit in terms of the dimensionless mass scale parameter $\bar{m}_0 = 0$ can be only achieved in one way: $m_0 \to 0$ at fixed Λ_{NP}, since at m_0 fixed the mass gap Λ_{NP} cannot go to infinity (it is either finite or zero, by definition [1, 2]).

Both solutions (2.13) and (2.14) are not, in principle, entire functions. The functions $A_0(x)$ and $B_0(c_0; x)$ have removable singularities at zero, i.e., they are finite at zero points. In addition, the dynamically generated quark mass function $B_0(c_0; x)$ also has the algebraic branch points at $x = c_0$ and at infinity (at fixed c_0). As in the general (non-chiral) case for the quark mass function (2.8) at $x = c$, these unphysical singularities are caused by the inevitable ghost contributions in the covariant gauges.

As was mentioned above, $A_0(x)$ automatically has a correct behavior at infinity (it does not explicitly depend on the constant of integration, since it was specified in order to get a regular at zero solution). In the PT limit ($\Lambda_{NP}^2 \to 0$) the constants of integration c_0, c and variable x go to infinity uniformly ($c_0, c, x \to \infty$), so the dynamically generated quark mass functions (2.8) and (2.14) identically vanish in this limit. Obviously, we have to keep the constants of integration c_0, c arbitrary but finite in order to obtain a regular at zero point solutions. The problem is that if $c_0, c = \infty$, then the corresponding solutions do not exist at all at any finite x, in particular at $x = 0$.

Concluding, let us note that the singular at zero point exact solutions of the system (2.12) also exist. It is easy to check that the exact solution $A_0(x) = (1/x^2)(1-x)$ automatically satisfies it. Substituting it into the Eq. (2.14), one obtains the exact solution for the dynamically generated quark mass function, which will be singular at zero point as well. However, similar to the ghost self-energy [2], the singular at zero solution for the quark propagator should be excluded from the consideration, since the smoothness properties of the test functions will be compromised in this case.

III. QUARK CONFINEMENT

In principle, it is possible to develop the calculation schemes in different modifications, which will give the solution of the ground system (2.7) step by step in powers of the light current quark masses as well as in the inverse powers of the heavy quark messes (see sections below). The important observation, however, is that the formal exact solution (2.8) exhibits the algebraic branch point at $x = c$, which completely *excludes a pole − type singularity* at any finite point on the real axis in the x-complex plane whatever solution for the $A(x)$ function might be. Thus the solution for the quark propagator cannot be presented as an expression having finally a pole-type singularity at any finite point $p^2 = -m^2$ (Euclidean signature), i.e.,

$$S(p) \neq \frac{const}{\hat{p} + m}, \tag{3.1}$$

certainly satisfying thereby the first necessary condition of quark confinement formulated at the fundamental quark level as the absence of a pole-type singularity in the quark propagator [1]. A quark propagator may or may not be an entire function, but in any case a pole-type singularity has to disappear. This is a general feature of quark confinement, which holds in any gauge (see our paper [1] and references therein).

In order to confirm this, let us assume the opposite to Eq. (3.1), i.e., that the quark propagator within our approach may have a pole-type singularity like the electron propagator has in quantum electrodynamics (QED) (see Eq. (3.4) below). In terms of the dimensionless quark form factors, defined in Eq. (2.6), this means that in the neighborhood of the assumed pole at $x = -m^2$ (Euclidean signature), they can be presented as follows:

$$\begin{aligned} A(x) &= \frac{1}{(x+m^2)^{\alpha}} \tilde{A}(x), \\ B(x) &= \frac{1}{(x+m^2)^{\beta}} \tilde{B}(x), \end{aligned} \tag{3.2}$$

where $\tilde{A}(x)$ and $\tilde{B}(x)$ are regular at a pole, while α and β are, in general, arbitrary with $Re\alpha, \beta \geq 0$. However, substituting these expansions into the system (2.7) and analyzing it in the neighborhood of the assumed pole, one can immediately conclude in that the self-consistent system for the quantities with tilde exists if and only if

$$\alpha = \beta = 0. \tag{3.3}$$

In other words, our system (2.7) does not admit a pole-type singularities in the quark propagator in complete agreement with the above-mentioned.

This point deserves a more detail discussion, indeed. The infrared (IR) asymptotic of the electron propagator in QED is [5] (Minkowski signature)

$$S(p) \sim \frac{1}{(p^2 - m^2)^{1+\beta}}, \tag{3.4}$$

where $\beta = \alpha(\xi - 3)/2\pi$ and here α is the renormalized charge and ξ is the gauge-fixing parameter. Thus instead of a simple pole, it has a cut whose strength can be varied by changing ξ. However, there is, in general, a pole-type singularity at the electron mass m, indeed, i.e., in QED there is no possibility to escape a pole-type singularity in the electron Green's function. Contrary to QED, our general solution (2.8) has no pole-type singularities, only the branch point at $x = c$, and the constant of integration c may, in general, depend implicitly on ξ. At the same time, it is obvious that the existence of a branch point itself does not depend explicitly on the gauge choice. Thus the absence of a pole-type singularity in QCD in the same way is gauge-invariant as the existence of a pole-type singularity at the electron mass in QED. This may be used indeed to differentiate QCD from QED and vice versa. The gauge invariance of the above-mentioned first necessary condition of quark confinement should be precisely understood in this sense.

The second sufficient condition formulated at the hadronic level as the existence of a discrete spectrum only (no continuum in the spectrum) [6] in the bound-state problems within the corresponding BS formalism is obviously beyond the scope of the present investigation. Let us only note here, that at nonzero temperature the bound-states will be dissolved (dehadronized), but the first necessary condition of the quark confinement criterion will remain valid, nevertheless. In other words, quarks at nonzero temperature, for example, in the quark-gluon plasma (QGP) [7], will remain off-shell objects, i.e., by increasing temperature they cannot be put on the mass-shell. Hence even in this case

they cannot be detected as physical particles (like electrons) in the asymptotic states. That is why it is better to speak about dehadronization phase transition in QGP rather than about deconfinement phase transition.

Let us make a few remarks in advance. The region $c \geq x$ can be considered as NP, whereas the region $c \leq x$ can be considered as the PT one (see next section). As was mentioned above, our solutions to the quark propagator are valid in the whole momentum range $[0, \infty)$. However, in order to calculate any physical observable from first principles (represented by the corresponding correlation function which can be expressed in terms of the quark propagator integrated out), it is necessary to restrict ourselves to the integration over the NP region $x \leq c$ ($x \leq c_0$) only. This guarantees us that the above-mentioned unphysical singularity (branch point at $x = c$ ($x = c_0$)) will not affect the numerical values of the physical quantities. Evidently, this is equivalent to the subtraction of the contribution in the integration over the PT region $x \geq c$ ($x \geq c_0$). Let us underline that at the hadronic level this is the necessary subtraction which should be only made (see discussion in the next section).

IV. DYNAMICAL BREAKDOWN OF CHIRAL SYMMETRY (DBCS)

From a coupled system of the differential equations (2.7) it is easy to see that this system

allows a chiral symmetry breaking solution only,

$$\bar{m}_0 = 0, \quad A(x) \neq 0, \; B(x) \neq 0 \tag{4.1}$$

and forbids a chiral symmetry preserving solution,

$$\bar{m}_0 = B(x) = 0, \quad A(x) \neq 0. \tag{4.2}$$

Thus any nontrivial solutions automatically break the γ_5 invariance of the quark propagator

$$\{\gamma_5, S(p)\} = i\gamma_5 2B(p^2) \neq 0, \tag{4.3}$$

and they therefore *certainly* lead to the spontaneous chiral symmetry breakdown at the fundamental quark level ($\bar{m}_0 = 0, B(x) \neq 0$, the dynamical quark mass generation). Let us emphasize that a measure of this breakdown is the twice dynamically generated quark mass, i.e., at a scale of twice dynamically generated quark mass a spontaneous chiral symmetry breakdown occurs. In principle, one can calculate the dynamical quark mass $B(p^2)$ at any finite point (at zero for regular functions as in our case) and at any covariant gauge. However, the important observation is that the above-mentioned definition of this measure is valid for any covariant gauge, and in this sense is gauge-invariant as it follows from the relation (4.3). In all previous investigations a chiral symmetry preserving solution (4.2) always exists. Here we do not distinguish between $B(p^2)$ and $\overline{B}(p^2)$ calling both dynamically generated quark mass functions, for simplicity.

A few preliminary remarks are in order. A nonzero dynamically generated quark mass function defined by conditions (4.1) and (4.3) is the order parameter of DBCS at the fundamental quark level. At the phenomenological level the order parameter of DBCS is the nonzero chiral quark condensate defined as the integral of the trace of the quark propagator in the chiral limit, i.e., (Euclidean signature, see Eq. (2.2))

$$< \bar{q}q >_0 = < 0|\bar{q}q|0 >_0 \sim i \int d^4p \; TrS(p), \tag{4.4}$$

up to unimportant (for our discussion) numerical factors. In terms of the dimensionless variables (2.6) it becomes

$$< \bar{q}q >_0 \sim -\Lambda^3_{NP} \int_0^\infty dx \; x \; B_0(x), \tag{4.5}$$

where for light quarks in the chiral limit $< \bar{q}q >_0 = < \bar{u}u >_0 = < \bar{d}d >_0 = < \bar{s}s >_0$, by definition.

It is worth emphasizing now that the phenomenological order parameter of DBCS - the chiral quark condensate - defined as the dynamically generated quark mass function $B_0(x)$ integrated out (4.5) might be in principle zero even when the mass function is definitely nonzero. Thus a nonzero dynamically generated quark mass is a much

more appropriate condition of DBCS than the chiral quark condensate. One can say that this is the first necessary condition of DBCS, while a nonzero chiral quark condensate is only the second sufficient one.

However, this is not the whole story yet. The problem is that the chiral quark condensate defined in Eq. (9.5) still contains the contribution in the integration over the PT region, say, $[y_0, \infty)$. In order to define correctly the chiral quark condensate in the INP QCD this contribution should be subtracted, i.e.,

$$< \bar{q}q >_0 \Longrightarrow < \bar{q}q >_0 + \Lambda_{NP}^3 \int_{y_0}^{\infty} dx \ x \ B_0(x) = -\Lambda_{NP}^3 \int_0^{y_0} dx \ x \ B_0(x). \tag{4.6}$$

If now the mass function $B_0(x)$ is really the NP solution of the corresponding quark SD equation, then this definition gives the quark condensate beyond the PT theory. In our case this is so, indeed. Moreover, it is easy to understand that in order to guarantee that the algebraic branch point at $x = c_0$ will not affect the numerical value of the quark condensate, the dimensionless scale y_0, separating the NP region from the PT one, should be identified with the constant of integration c_0. Thus in our case the truly NP chiral quark condensate becomes

$$< \bar{q}q >_0 \sim -\Lambda_{NP}^3 \int_0^{c_0} dx \ x \ B_0(c_0; x), \tag{4.7}$$

i.e., the truly NP dynamically generated quark mass function is integrated out over the NP region as well. So there is not even a bit of the PT information in this definition (all types of the PT contributions have been already subtracted in Eq. (4.7)). The point of subtraction at the hadronic level is determined by the constant of the integration at the quark level (branch point). Moreover, it depends on the mass gap Λ_{NP} and not on the arbitrary mass scales 1 GeV, 2 GeV, etc. In the PT limit $\Lambda_{NP}^2 \to 0$ the quark condensate goes to zero as it should be, by definition ($B_0(c_0; x)$ identically vanishes in this case as well since, let us remind, $c_0, x \to \infty$ uniformly). Thus in our approach the chiral quark condensate itself has a physical meaning.

The actual calculation of the chiral quark condensate as well as all other chiral QCD parameters will be given in the subsequent paper, where the Goldstone sector of QCD will be analytically investigated and numerically evaluated (for preliminary, however, numerical results see our papers [8, 9]).

V. NONZERO CURRENT QUARK MASSES. LIGHT QUARKS

To derive the explicit solutions for the quark propagator in the general (non-chiral) case, it is convenient to start from the ground system (2.7) itself, and rewrite it as follows:

$$\begin{aligned} xA' + (2 + x)A + 1 &= -\bar{m}_0 B, \\ 2BB' + 3A^2 + 2B^2 &= 2\bar{m}_0 AB, \end{aligned} \tag{5.1}$$

where, let us remind, $A \equiv A(x)$, $B \equiv B(x)$, and here the prime denotes the derivative with respect to the Euclidean dimensionless momentum variable x. The dimensionless current quark mass $\bar{m}_0$ is defined as $\bar{m}_0 = m_0/\Lambda_{NP}$. We are interested in the solutions which are regular at zero and asymptotically approach the free quark case at infinity. Because of our parametrization of the quark propagator (2.2), its asymptotic behavior has to be determined as follows (Euclidean metrics):

$$A(x) \sim_{x\to\infty} -\frac{1}{x}, \qquad B(x) \sim_{x\to\infty} -\frac{\bar{m}_0}{x}. \tag{5.2}$$

The ground system (5.1) is very suitable for numerical calculations.

Let us now develop an analytical formalism which makes it possible to find solution of the ground system (5.1) step by step in powers of the light current quark masses, the so-called chiral perturbation theory at the fundamental quark level. For this purpose let us present the quark propagator form factors A and B as follows:

$$\begin{aligned} A(x) &= \sum_{n=0}^{\infty} \bar{m}_0^n A_n(x), \\ B(x) &= \sum_{n=0}^{\infty} \bar{m}_0^n B_n(x), \end{aligned} \tag{5.3}$$

and for light current quark masses one has $\bar{m}_0^{(u,d,s)} \ll 1$ (let us note in advance that this estimate is justified, indeed). Substituting these expansions into the ground system (5.1) and omitting some tedious algebra, one obtains

$$\begin{aligned} xA_0'(x) + (2+x)A_0(x) + 1 &= 0, \\ 2B_0(x)B_0'(x) + 3A_0^2(x) + 2B_0^2(x) &= 0, \end{aligned} \tag{5.4}$$

and for $n = 1, 2, 3, ...,$ one has

$$\begin{aligned} xA_n'(x) + (2+x)A_n(x) &= -B_{n-1}(x), \\ 2P_n(x) + 3M_n(x) + 2Q_n(x) &= 2N_{n-1}(x), \end{aligned} \tag{5.5}$$

where

$$\begin{aligned} P_n(x) &= \sum_{m=0}^{n} B_{n-m}(x)B_m'(x), \\ M_n(x) &= \sum_{m=0}^{n} A_{n-m}(x)A_m(x), \\ Q_n(x) &= \sum_{m=0}^{n} B_{n-m}(x)B_m(x), \\ N_n(x) &= \sum_{m=0}^{n} A_{n-m}(x)B_m(x). \end{aligned} \tag{5.6}$$

Is is obvious that the system (5.4) describes the ground system (5.1) in the chiral limit ($\bar{m}_0 = 0$), and thus coincides with the system (2.12). Its exact solution is given by Eqs. (2.13) and (2.14). The first nontrivial correction in powers of small $\bar{m}_0$ is determined by the following system

$$\begin{aligned} xA_1' + (2+x)A_1 &= -B_0, \\ (B_1B_0' + B_0B_1') + 3A_0A_1 + 2B_0B_1 &= A_0B_0, \end{aligned} \tag{5.7}$$

where we omit the dependence on the argument x, for simplicity. In a similar way can be found the system of equations to determine terms of order $\bar{m}_0^2$ in the solution for the quark propagator and so on.

Let us present a general solution of the first of Eqs. (5.5) as

$$A_n(x) = -x^{-2}e^{-x} \int_0^x dx' \ x'e^{x'} B_{n-1}(x'), \tag{5.8}$$

which is always regular at zero, since all $B_n(x)$ are regular as well. The advantage of the developed chiral perturbation theory at the fundamental quark level is that each correction in the powers of small current quark masses is determined by the corresponding system of equations which can be formally solved exactly.

Let us write down the system of solutions approximating the light quark propagator up to first corrections, i.e.,

$$\begin{aligned} A(x) &= A_0(x) + \bar{m}_0 A_1(x) + ..., \\ B(x) &= B_0(x) + \bar{m}_0 B_1(x) + ... \end{aligned} \tag{5.9}$$

This system is

$$A_0(x) = x^{-2}(1 - x - e^{-x}), \quad A_0(0) = -\frac{1}{2}, \tag{5.10}$$

$$B_0^2(x) = 3e^{-2x} \int_x^{c_0} dx' \ e^{2x'} A_0^2(x'). \tag{5.11}$$

And

$$A_1(x) = -x^{-2}e^{-x}\int_0^x dx'\ x'e^{x'}B_0(x'), \tag{5.12}$$

$$B_1(x) = e^{-2x}B_0^{-1}(x)\int_{c_1}^x dz\ e^{2z}A_0(z)[B_0(z) - 3A_1(z)]. \tag{5.13}$$

In physical applications we also need $B^2(x)$, so we have

$$\begin{aligned} B^2(x) &= B_0^2(x) + 2\bar{m}_0 B_0(x)B_1(x) + ..., \\ &= B_0^2(x) + 2\bar{m}_0 e^{-2x}\int_{c_1}^x dz\ e^{2z}A_0(z)[B_0(z) - 3A_1(z)] + ..., \end{aligned} \tag{5.14}$$

and the relation between constants of integration c_0 and c_1 remains, in general, arbitrary. However, there exists a general restriction, namely $B^2(x) \geq 0$ and real, which may lead to some bounds for the constants of integration, however, that is $x \leq c_0$ always remains valid.

VI. NONZERO CURRENT QUARK MASSES. HEAVY QUARKS

For heavy quarks it makes sense to replace $m_0 \to m_Q$, i.e., to put $\bar{m}_Q = m_Q/\Lambda_{NP}$, and rewrite thus the system of equations (5.1) as follows:

$$\begin{aligned} xA' + (2+x)A + 1 &= -\bar{m}_Q B, \\ 2BB' + 3A^2 + 2B^2 &= 2\bar{m}_Q AB, \end{aligned} \tag{6.1}$$

where, let us remind, $A \equiv A(x)$, $B \equiv B(x)$, and here the prime denotes the derivative with respect to the Euclidean dimensionless momentum variable x. We are again interested in the solutions which are regular at zero and asymptotically approach the free quark case at infinity (5.2), on account of the above-mentioned replacement $m_0 \to m_Q$.

In this case it is convenient to find a solution for heavy quark form factors A and B in the form of the corresponding expansions, namely

$$\begin{aligned} \bar{m}_Q^2 A(x) &= \sum_{n=0}^{\infty} \bar{m}_Q^{-n} A_n(x), \\ \bar{m}_Q B(x) &= \sum_{n=0}^{\infty} \bar{m}_Q^{-n} B_n(x), \end{aligned} \tag{6.2}$$

and for heavy quark masses we have $\bar{m}_Q^{(c,b,t)} \gg 1$, i.e., the inverse powers are small (let us note in advance that this estimate is justified, indeed). In terms of the dimensionless mass scale parameter $\bar{m}_Q$ the heavy quarks large mass limit can be formally achieved by the two ways: $m_Q \to \infty$ at fixed Λ_{NP} and at m_Q fixed, while $\Lambda_{NP} \to 0$.

Substituting these expansions into the first equation of the ground system (6.1) and omitting some tedious algebra, one obtains

$$B_0(x) = -1, \quad B_1(x) = 0, \tag{6.3}$$

and

$$xA_n'(x) + (2+x)A_n(x) = -B_{n+2}(x), \quad n = 0, 1, 2, 3, ... \tag{6.4}$$

In the same way, by equating terms at equal powers in the inverse of heavy quark masses, from second of the equations of the ground system (6.1), one obtains

$$\begin{aligned} P_0(x)+Q_0(x)-N_0(x) &= 0, \\ P_1(x)+Q_1(x)-N_1(x) &= 0. \end{aligned} \tag{6.5}$$

and

$$P_{n+2}(x)+Q_{n+2}(x)-N_{n+2}(x) = -\frac{3}{2}M_n(x), \quad n=0,1,2,3,..., \tag{6.6}$$

where $P_n(z),\ M_n(z),\ Q_n(z),\ N_n(z)$ are again given formally by Eqs. (5.6). Solving these equations, one obtains

$$\begin{aligned} A_0(x) &= B_0(x) = -1, \\ A_1(x) &= B_1(x) = 0, \end{aligned} \tag{6.7}$$

and

$$\begin{aligned} xA_n'(x)+(2+x)A_n(x) &= -B_{n+2}(x), \\ P_{n+2}(x)+Q_{n+2}(x)-N_{n+2}(x) &= -\frac{3}{2}M_n(x), \quad n=0,1,2,3,... \end{aligned} \tag{6.8}$$

It is possible to show that all odd terms are simply zero, i.e.,

$$A_{2n+1}(x) = B_{2n+1}(x) = 0, \quad n=0,1,2,3,... \tag{6.9}$$

The explicit solutions for a few first nonzero terms are

$$A_0(x) = B_0(x) = -1. \tag{6.10}$$

$$\begin{aligned} A_2(x) &= x+\frac{3}{2}, \\ B_2(x) &= x+2. \end{aligned} \tag{6.11}$$

$$\begin{aligned} A_4(x) &= -x^2-\frac{1}{2}x-6, \\ B_4(x) &= -x^2-\frac{9}{2}x-3. \end{aligned} \tag{6.12}$$

Thus our solutions for the heavy quark form factors look like

$$A(x) = \frac{1}{\bar{m}_Q^2}\sum_{n=0}^{\infty}\bar{m}_Q^{-n}A_n(x) = -\frac{1}{\bar{m}_Q^2}+\frac{x}{\bar{m}_Q^4}-\frac{x^2}{\bar{m}_Q^6}+...+D_A(x), \tag{6.13}$$

where

$$D_A(x) = \frac{3}{2\bar{m}_Q^4}-\frac{x+12}{2\bar{m}_Q^6}+... \tag{6.14}$$

And

$$B(x) = \frac{1}{\bar{m}_Q}\sum_{n=0}^{\infty}\bar{m}_Q^{-n}B_n(x) = -\frac{1}{\bar{m}_Q}+\frac{x}{\bar{m}_Q^3}-\frac{x^2}{\bar{m}_Q^5}+...+D_B(x), \tag{6.15}$$

where

$$D_B(x) = \frac{2}{\bar{m}_Q^3} - \frac{9x+6}{2\bar{m}_Q^5} + ... \tag{6.16}$$

Summing up, one obtains

$$\begin{aligned} A(x) &= -\frac{1}{x+\bar{m}_Q^2} + D_A(x), \\ B(x) &= -\frac{\bar{m}_Q}{x+\bar{m}_Q^2} + D_B(x). \end{aligned} \tag{6.17}$$

In terms of the Euclidean dimensionless variables (2.6), the quark propagator (2.2) is

$$iS(x) = \hat{x}A(x) - B(x). \tag{6.18}$$

Using our solutions, obtained above, it can be written down as follows:

$$iS_h(x) = iS_0(x) + \hat{x}D_A(x) - D_B(x), \tag{6.19}$$

where $iS_0(x)$ is nothing but the free quark propagator with the substitution $m_0 \to m_Q$, i.e.,

$$iS_0(x) = -\frac{\hat{x} - \bar{m}_Q}{x+\bar{m}_Q^2}. \tag{6.20}$$

Since $\hat{x}D_A(x) - D_B(x)$ is of the order $\bar{m}_Q^{-3}$, then the heavy quark propagator (6.19) becomes

$$iS_h(x) = iS_0(x) + O(1/\bar{m}_Q^3), \tag{6.21}$$

which means that our solution for the heavy quark propagator is reduced to the free quark propagator up to the terms of the order $1/\bar{m}_Q^3$.

A. Heavy quark-gluon vertex

For further purpose, it is instructive to go back to the dimensional momentum variable p in the heavy quark propagator, i.e., to rewrite Eq. (6.21) up to the terms of the order $1/m_Q^3$ as follows:

$$S_h(p) = i\frac{\hat{p} - m_Q}{p^2 + m_Q^2}. \tag{6.22}$$

It is easy to derive that quantities defined in Eqs. (2.4) become

$$\begin{aligned} \overline{A}(p^2) &= -1, \\ \overline{B}(p^2) &= -m_Q, \\ E(p^2) &= (p^2 + m_Q^2)^{-1}, \end{aligned} \tag{6.23}$$

so that for the corresponding form factors (2.5) one gets

$$\begin{aligned} F_1(p^2) &= \frac{1}{2}, \\ F_2(p^2) &= F_4(p^2) = \frac{1}{2m_Q}, \\ F_3(p^2) &= 0. \end{aligned} \tag{6.24}$$

Thus the explicit expression for the corresponding heavy quark-gluon vertex (2.1) becomes

$$\Gamma_\mu(p,0) = \frac{1}{2}\Big[\gamma_\mu + \frac{1}{m_Q}p_\mu + \frac{1}{m_Q}\hat{p}\gamma_\mu\Big]. \tag{6.25}$$

It is worth noting that an overall numerical factor 1/2 in this vertex is due to the second term in the ST identity (1.1), i.e., it is due to the fact that our initial identity is the ST one and not QED-type in which the first term is to be only presented.

Concluding, a few remarks are in order. Starting from the expansion (6.2) for the $A(x)$ function and using exact Eqs. (2.10) and (2.11), one obtains the same free quark propagator solution (6.17) for it. In other words, the straightforward solution of the initial system (6.1) coincides with the exact solution for the $A(x)$ function, indeed. Unfortunately, things are not so simple for the heavy quark mass function $B(x)$. Substituting the free quark propagator solution (6.17) for the $A(x)$ function into the exact integral (2.8), on account of the exact relation (2.9), one obtains the expression which is by no means the solution of the free quark propagator for the $B(x)$ function shown in Eq. (6.17). It can be only reduced to it in the direct $\bar{m}_Q \to \infty$ limit. Thus such obtained solution is a new one for the heavy quark mass function, and it is left to be investigated elsewhere.

VII. CONCLUSIONS

We have investigated a closed system of equations for the quark propagator obtained on the basis of our approach to low-energy QCD which we call INP QCD. One of its general features is the subtractions of all types and at all levels the PT contributions ("contaminations") from QCD. The above-mentioned system of equations consists of the quark SD equation itself, which is complemented by the quark ST identity for the corresponding quark-gluon vertex. This system is free of all types of the PT contributions at the fundamental quark-gluon-ghost level. Moreover, it is manifestly gauge-invariant, i.e., does not depend explicitly on the gauge-fixing parameter. It depends explicitly only on the mass gap responsible for the truly NP dynamics in the QCD ground state. For the dynamically generated quark mass function $B(p^2)$ it admits an exact formal solution in terms of the $A(p^2)$ function, see Eq. (2.8). The $A(p^2)$ function itself should satisfy the nonlinear differential equation (2.11). This system of equations can be solved exactly in the chiral limit. In the case of nonzero light quark masses, we develop an analytical formalism, the so-called chiral perturbation theory at the fundamental quark level, which allows one to find a solution for the quark propagator in powers of the light quark masses. We also develop an analytical formalism, which allows us to find solution for the quark propagator in the inverse powers of heavy quark masses. For the first time it has been theoretically justified the use of the free quark propagator for heavy quarks. We have established that this is possible even up to terms of the order $1/m_Q^3$. Thus our solution automatically possesses the heavy quark spin-flavor symmetry. However, we would like to make once more perfectly clear the two main properties of our solution for the quark propagator.

A. Quark confinement

One of the most important observations is that a formal exact solution (2.8) for the dynamically generated quark mass function has a branch point at $x = c$, which completely excludes a pole-type singularity for the quark propagator similar to the singularity which has electron propagator in QED. As mentioned above, the quark confinement criterion in QCD consists of the two conditions.

I. The first necessary condition should be formulated at the fundamental quark-gluon level as the absence of a pole-type singularity in the quark propagator (see Eq. (3.1) and discussion therein).

II. The second sufficient condition should be formulated at the hadronic level as the existence of a discrete spectrum only (no continuum in the spectrum) in the bound-state problems.

Though our solution for the quark propagator is formally valid in the whole energy/momentum range, the integration out over the quark degrees of freedom should be obviously made up to the branch point $x = c$ in order to prevent the quark propagator to be pure imaginary in the region $x > c$. This is in agreement with the first necessary condition. In this connection let us remind that nonconfining electron propagator always has an imaginary part. In fact, the existence of the branch point explicitly shows where the PT contributions are to be exactly subtracted when the quarks degrees of freedom should be integrated out. For the gluon degrees of freedom integrated out we already know the exact point of the subtraction [10, 11].

At the macroscopic, hadronic level the linear rising potential interpretation of confinement becomes relevant for bound states between heavy quarks only. In this case the full vertex can be approximated by its point-like counterpart (see Eq. (6.25) up to the terms of the order $1/m_Q$). Saturating further the Wilson loop by the one-gluon exchange diagram with the dominant $(q^2)^{-2}$ behavior for the gluon propagator [1, 2], one precisely obtains the Wilson criterion of quark confinement–area law [12, 13], or equivalently the linear rising potential between heavy quarks. In this case confinement looks more like that of the Schwinger model [14] of two-dimensional QED and of the 't Hooft model [6] of two-dimensional axial gauge QCD, where non-Abelian degrees of freedom have been eliminated by the choice of the gauge.

Thus, the Wilson criterion of quark confinement and consequently its linear rising potential interpretation based on lattice gauge theory is valid only for heavy quarks, and, in general, is inadequate for the continuous theory [15], indeed. At the same time, a quark confinement criterion formulated above within the continuous theory is valid for any quarks (light or heavy, does matter). To analyze quark confinement in terms of the analytic properties of the quark propagator is much more relevant. This is reflected in the above-formulated its general criterion. Heavy quarks demonstrate some special properties, such as spin-flavor symmetry, linear rising potential, a possibility to use the free quark propagators for them, etc. All this substantially simplifies the investigation of the bound-states consisting of them.

Concluding, let us note that confinement has been proven for two-dimensional covariant gauge QCD (i.e., taking into account non-Abelian degrees of freedom) in our papers [16].

B. DBCS

The second important observation is that the γ_5 invariance of the quark propagator is broken, for sure within our approach, see Eq. (4.3). As emphasized above, this means the dynamical quark mass generation, and thus dynamical (or equivalently spontaneous) breakdown of chiral symmetry. For the existence of this phenomenon the two conditions are also required.

I. At the fundamental quark level the first necessary condition is to be formulated as the absence of a chiral symmetry preserving solution for the quark propagator, while a chiral symmetry breaking solution is only allowed (see Eqs. (4.2) and (4.1)).

II. At the phenomenological level the second sufficient condition is to be formulated as the existence of the nonzero chiral quark condensate, Eq. (4.7).

The only problem with the nonzero quark condensate is that it should be correctly calculated as explained above, i.e., the subtraction of all types and at all levels of the PT contributions should be correctly done. This also assumes that it should depend on the mass gap responsible for the NP dynamics in the QCD ground state. One can assign a physical meaning to it now (since it does not depend on the arbitrary scale) as the quantity which measures the density of quark degrees of freedom in the vacuum [17]. Precisely in this way the chiral quark condensate was calculated within our approach (for preliminary numerical results see our papers [8, 9]).

C. INP QCD

Let us briefly formulate some general features of INP QCD as a theory of QCD at low energies.

(a). INP QCD is defined by the subtraction from QCD of all types and at all levels of the PT contributions ("contaminations").

(b). First necessary subtraction should be made at the fundamental gluon propagator level. The confining gluon propagator has been obtained by the direct iteration solution of the gluon SD equation in the presence of a mass gap.

(c). Free from ghost complications because of the above-mentioned subtractions in the ghost and quark-gluon sectors, though the information on the quark degrees of freedom containing in the ghost-quark scattering kernel has been self-consistently taken into account.

(d). Implies quark confinement.

(e). Implies DBCS.

(f). Has a uniquely defined physical mass gap responsible for color confinement, DBCS and all other NP phenomena.

(g). Effectively short-range theory despite gluons remain massless.

(h). The chiral limit physics (i.e., the Goldstone sector) can be exactly evaluated.

(i). The point of the subtraction of the PT contributions at the hadronic (macroscopic) level is exactly known. This is the branch point $x = c/x = c_0$ in the general/chiral solution for the dynamically generated quark mass function

(2.8)/(2.14). Thus in this theory the NP (soft momenta) region is exactly separated from the PT (hard momenta) one.

It is worth discussing this feature of INP QCD in more detail. The existence of the point at which the PT tails should be exactly subtracted makes it possible to establish the space of the smooth test functions, consisting of the quark propagator and the corresponding quark-gluon vertex. In this space our generalized function (the confining gluon propagator) becomes a continuous linear functional. It is a linear topological space, denoted as $K(c)$ (in the chiral limit as $K(c_0)$), consisting of infinitely differentiable functions having compact support in the region $x \le c$, i.e., test functions which vanish outside the interval $x \le c$ (in the chiral limit $x \le c_0$). It is well known that this space can be identified with a complete countably normed space, which is at the same time a linear metric space, and the topology defined by the metric is equivalent to the original topology [18]. Thus the above-mentioned subtraction of all kind of the PT contributions at the hadronic level becomes not only physically well justified but mathematically confirmed by the distribution theory [18] as well. On the other hand, if one defines the space of the smooth test functions as above, which is necessary because of the existence of the branch point, then the subtractions of the PT tails become inevitable, indeed.

Concluding, a few remarks are in order. As underlined above, INP QCD makes it possible to calculate the physical obsevables/processes in low-energy QCD from first principles and in a self-consistent way. For this purpose what one should mainly do is to express the S matrix element of any physical quantity in terms of the corresponding loop integral(s) over the derived confining quark propagator and the corresponding bound-state amplitudes. These loop integrals are integrals over the finite volume in which only the pure NP excitations and fluctuations of the virtual transversal gluon/ghost and quark fields are important. Due to the character of the above-established space of the smooth test functions, the fluctuations of the PT character, origin and magnitude have been totally "washed out" from this volume, and it remains free of them for ever. For preliminary numerical calculations see, for example, the above-mentioned Refs. [8, 9]).

The necessity to integrate up to the branch point, i.e., over the finite range only, points out the existence of something like the Gribov horizon [19, 20], established in the functional space. Our "horizon" exists in the momentum space, which is much simper than the functional space. On the other hand, it has a clear physical meaning as separating the NP "world" from the PT one in the true QCD vacuum. It is well known that to solve the problem of an ambiguity in the gauge-fixing of non-Abelian gauge fields (the so-called Gribov ambiguity (uncertainty), which results in Gribov copies and vice-versa), he proposed to integrate over the finite range in the functional space of non-Abelian gauge fields, which consists in integrating only over the fields for which the Fadeev-Popov determinant is positive, introducing thus the above-mentioned horizon. Apparently, its existence in different spaces (functional, where the Gribov copies problem is explicitly present, momentum, where it is implicitly present, etc.) is inevitable at the macroscopic level. Otherwise this problem will plague the dynamics of any essential non-linear gauge systems at all levels. Within our approach the solution of the Gribov ambiguity problem in the gauge-fixing of non-Abelian gauge fields at the fundamental (microscopic) gluon propagator level is discussed in Ref. [1].

After performing the ultraviolet (UV) renormalization program for the mass gap [1] and making all necessary subtractions of the PT tails, this theory is evidently UV finite, i.e., free of the UV divergences. Also, after performing the infrared (IR) renormalization program for the mass gap [1], this theory becomes IR renormalizable, i.e., free of severe IR singularities, expressed in terms of the IR regularization parameter [1, 2]. This is true at least for the considered sectors of the theory [2] in order to derive the confining quark propagator, which is needed for the above-mentioned calculations from first principles. The general IR multiplicative renormalizability of this theory is a rather technical issue and is left to be proven elsewhere. It is beyond the scope of the present investigation.

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APPENDIX A: HEAVY QUARKS SPIN-FLAVOR SYMMETRY

In order to investigate the heavy quarks symmetries ([21] and references therein) it is convenient to consider the heavy quark propagator (6.22) and the corresponding heavy quark-gluon vertex (6.25) in Minkowski space, since the investigation of the above-mentioned symmetries includes hadron degrees of freedom (see below). In Minkowski space they become

$$S_h(p) = i\frac{\hat{p} + m_Q}{p^2 - m_Q^2} \tag{A1}$$

and

$$\Gamma_\mu(p,0) = \frac{1}{2}\Big[\gamma_\mu + \frac{1}{m_Q}p_\mu - \frac{1}{m_Q}\hat{p}\gamma_\mu\Big]. \tag{A2}$$

Let us explicitly show here that our solution for the quark propagator in this case possesses the heavy quark flavor symmetry, indeed. We will show that the quark propagator to leading order in the inverse powers of the heavy quark mass will not depend on it, i.e., it is manifestly flavor independent to the leading order of this expansion. For this purpose, a standard heavy quark momentum decomposition should be used, namely

$$p_\mu = m_Q v_\mu + k_\mu, \tag{A3}$$

where v is the four-velocity with $v^2 = 1$. It should be identified with the four-velocity of the hadron. The "residual" momentum k is of dynamical origin. Substituting this decomposition into the Eq. (A1), and taking into account only to leading order terms in the inverse powers of m_Q, one finally obtains

$$S_h(v,k) = \frac{i}{v \cdot k}P_+ + O(\frac{1}{m_Q}), \tag{A4}$$

which is exactly the heavy quark propagator [21]. Thus our propagator does not depend on m_Q to leading order in the heavy quark mass limit, $m_Q \to \infty$, i.e., in this limit it possesses the heavy quark flavor symmetry, indeed. The operator P_+ is a positive-energy projection operator, which satisfies the following relations

$$P_\pm = \frac{1}{2}(1 \pm \hat{v}), \quad P_\pm^2 = P_\pm, \quad P_\pm P_\mp = 0. \tag{A5}$$

At the same time, the heavy-quark-gluon vertex (A2) in this limit does not depend explicitly on the "residual" momentum k, i.e.,

$$\Gamma_\mu(v,0) = \frac{1}{2}\Big[\gamma_\mu + v_\mu - \hat{v}\gamma_\mu + O_\mu(1/m_Q)\Big]. \tag{A6}$$

Let us now present some useful relations, namely

$$P_+\gamma_\mu P_+ = P_+ v_\mu P_+, \quad P_+\hat{v}\gamma_\mu P_+ = P_+ v_\mu, \tag{A7}$$

which can be easily checked. Because of these relations, the heavy-quark-gluon vertex effectively becomes

$$\Gamma_\mu(v,0) = v_\mu - \frac{1}{2}P_+ v_\mu + O_\mu(1/m_Q), \tag{A8}$$

i.e., the interactions of the heavy quark with light degrees of freedom does not depend on its spin as well in the heavy quark large mass limit (the dependence on the γ-matrix disappears). This is a direct manifestation of the heavy quark spin symmetry up to terms of the order $1/m_Q$. The existence of the second term in this vertex is due to fact that our ST identity is not trivial one, i.e., it is not QED-type as mentioned above. Otherwise the only first term would survive (contribute). Taking further into account the relation

$$P_+P_+v_\mu P_+ = P_+v_\mu P_+, \tag{A9}$$

which is valid because of the second of relations presented in Eq. (A7), the coupling of a heavy quark to gluons (A6) can be effectively simplified further to

$$\Gamma_\mu(\upsilon, 0) = \frac{1}{2}\upsilon_\mu + O_\mu(1/m_Q), \tag{A10}$$

and again the coefficient 1/2 is a manifestation that our initial ST identity was not of QED-type one.

Concluding, let us note that the general solution (2.8) does not demonstrate the principal difference in the analytical structure of its solutions for light and heavy quarks. At the fundamental quark level the heavy quark mass limit is not Lorentz covariant, and therefore for numerical calculations we will use rather Eq. (2.8) than Eq. (6.17). In order to analyze a possible symmetries of the interaction of heavy quarks with light degrees of freedom, we will go to the heavy quark large mass limit ($m_Q \to \infty$) at the final stage only within our approach.

[1] V. Gogokhia, hep-ph/0702066.
[2] V. Gogokhia, arXiv:0704.1745v1 [hep-ph].
[3] V.Sh. Gogokhia, Phys. Lett. B 224 (1989) 177.
[4] V.Sh. Gogokhia, Phys. Rev. D 40 (1989) 4157; ibid., 41 (1990) 3279.
[5] N.N. Bogoliubov, D. V. Shirkov, Introduction to the Theory of Quantised Fields (Interscience Publishers, NY, 1959).
[6] G. 't Hooft, Nucl. Phys. B 75 (1974) 461.
[7] A.S. Kronfeld, hep-ph/0209321.
[8] V. Gogohia, Gy. Kluge, M. Prisznyak, Phys. Lett. B 368 (1996) 221; ibid., 378 (1996) 385.
[9] V. Gogohia, H. Toki, T. Sakai, Gy. Kluge, Int. Jour. Mod. Phys. A 15 (2000) 45.
[10] V. Gogokhia, hep-ph/0508224.
[11] V. Gogohia, Gy. Kluge, Phys. Rev. D 62 (2000) 076008.
[12] K.G. Wilson, Phys. Rev. D 10 (1974) 2445.
[13] M. Bunder, Phys. Rep., 75 (1981) 205.
[14] W. Marciano, H. Pagels, Phys. Rep. C 36 (1978) 137.
[15] A.M. Polyakov, Nucl. Phys. B 486 (1997) 23.
[16] V. Gogohia, Phys. Lett. B 531 (2002) 321;
V. Gogokhia, Gy. Kluge, Phys. Rev. D 66 (2002) 056013.
[17] M. A. Shifman, A. I. Vainshtein, V. I. Zakharov, Nucl. Phys. B 147 (1979) 385, 448.
[18] I.M. Gel'fand, G.E. Shilov, Generalized Functions, (Academic Press, NY, 1968) Vol. II.
[19] V.N. Gribov, Nucl. Phys. B 139 (1978) 1.
[20] Y.L. Dokshitzer, D.E. Kharzeev, hep-ph/0404216.
[21] M. Nuebert, Phys. Rep. 245 (1994) 259.